TONBRIDGE SCHOOL

Test for Entrance into Year 12 in September 2015

Maths

Name:……

School:……

Answer **all** questions in Section A and

either question 12 or 13 in Section B.

Do all your workings in the spaces provided.

Marks for Section A: 60

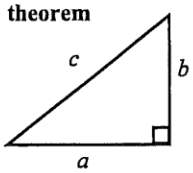
Marks for Section B: 20

Time allowed : 75 minutes.

You are allowed to use a calculator in this exam.

A list of useful formulae will be found on page 2.

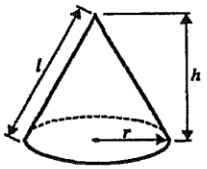
Pythagoras' theorem



$a^2+b^2=c^2$

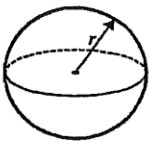
Volume of cone = $\frac{1}{3}\pi r^2 h$

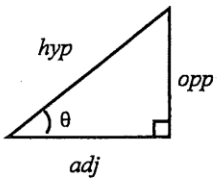
Curved surface area of cone = $\pi r l$



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$





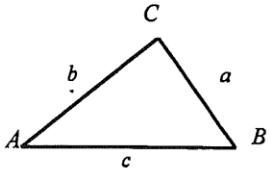
adj = hyp × cos θ
opp = hyp × sin θ
opp = adj × tan θ

or $\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

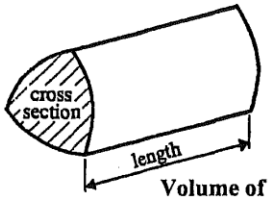
In any triangle ABC



Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

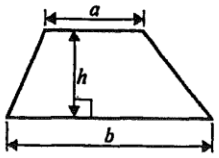
Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

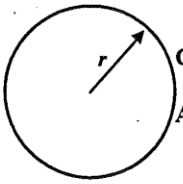
Area of triangle = $\frac{1}{2}ab\sin C$



Volume of prism = area of cross section × length

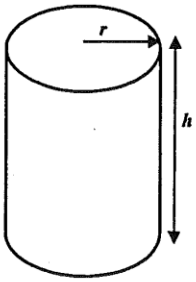
Area of trapezium = $\frac{1}{2}(a+b)h$





Circumference of circle = $2\pi r$

Area of circle = πr^2



Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi rh$

The quadratic equation

The solutions of $ax^2+bx+c=0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Section A :
Answer all questions in this section

1. Solve the simultaneous equations

$$5x + 4y = 3$$
$$x - 2y = 2$$

$x =$ ……
$y =$ ……
(Total 3 marks)

2. Make g the subject of the formula $T = 2\pi\sqrt{\frac{l}{g}}$

$g =$ ……..
(Total 4 marks)

3. (a) Multiply out the brackets and simplify your answer

$$(2x + 3)(2x - 5)$$

……..

(2)

(b) Factorise fully $3x^2 - 12$

……..

(2)

(c) Simplify $\frac{3x^2y \times 4xy^3}{6xy^5}$

…….

(3)

(Total 7 marks)

4. If $1 < a < 10, 1 < b \leq 9$ and $ab > 10$ then find, giving your answers in standard form, in terms of a and b;

(i) $(a \times 10^{15}) \times (b \times 10^{16})$

.....

(3)

(ii) $(a \times 10^{15}) + (b \times 10^{16})$

.....

(3)

(Total 6 marks)

5.

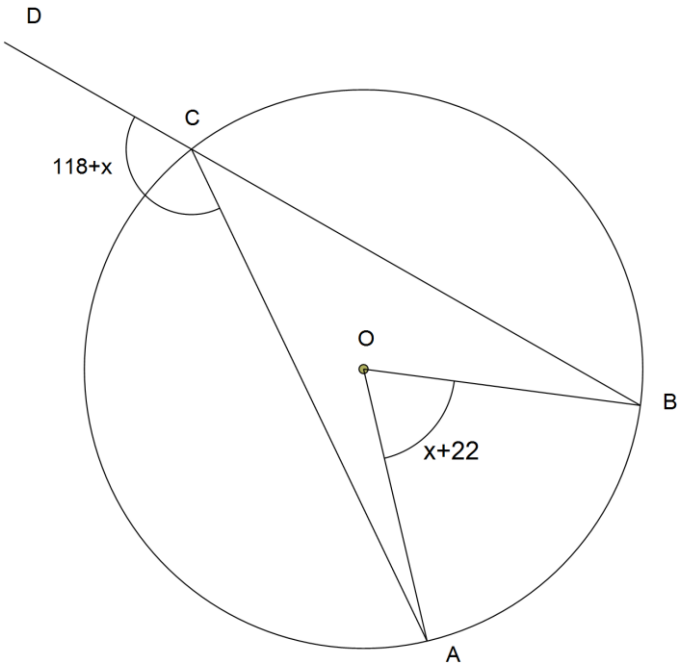


Diagram NOT drawn accurately

A, B and C are points on a circle, centre O.
BCD is a straight line.
Find the value of x.

$x =$ ……..

(Total 4 marks)

6. I roll three fair dice.

Calculate the probability that I throw

(i) A total score of 3

……..

(ii) A total score of 6

……

(Total 6 marks)

7. (i) Factorise $3x^2 + 5x - 2$

……..

(2)

(ii) Solve $2x^2 - 3x - 4 = 0$, giving your answers to 3 SF.

……..

(3)

(iii) Simplify

$$\frac{\frac{1}{x} - 2}{4 - \frac{1}{x^2}}$$

……

(3)

(Total 8 marks)

8. (i) Write $\frac{2}{2x-1} - \frac{1}{x+1}$ as a single algebraic fraction.

.....

(3)

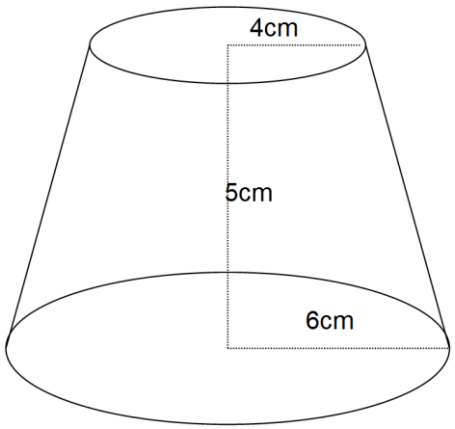
(ii) Hence solve $\frac{2}{2x-1} - \frac{1}{x+1} = \frac{1}{5-x}$.

.....

(4)

(Total 7 marks)

9.



The diagram shows a frustum of a cone, which is a cone with its top taken off.
The base radius is 6cm, the top radius is 4cm and the height is 5cm.
Find the volume of the frustum.

…… cm^3

(Total 5 marks)

10.

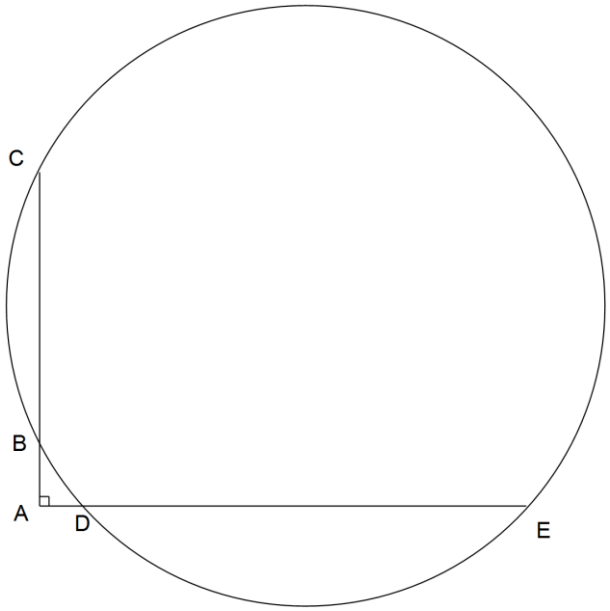


Diagram not

drawn accurately

The diagram shows a circle which passes through points B, C, E and D, and two straight lines ABC and ADE which intersect at right angles at A.
AD=4cm, DE=40cm and BC=14cm.

(i) Find the radius of the circle.

$r =$.....cm

(3)

(ii) Hence, or otherwise, find the length AB.

$AB =$.....cm

(2)

(Total 5 marks)

11.

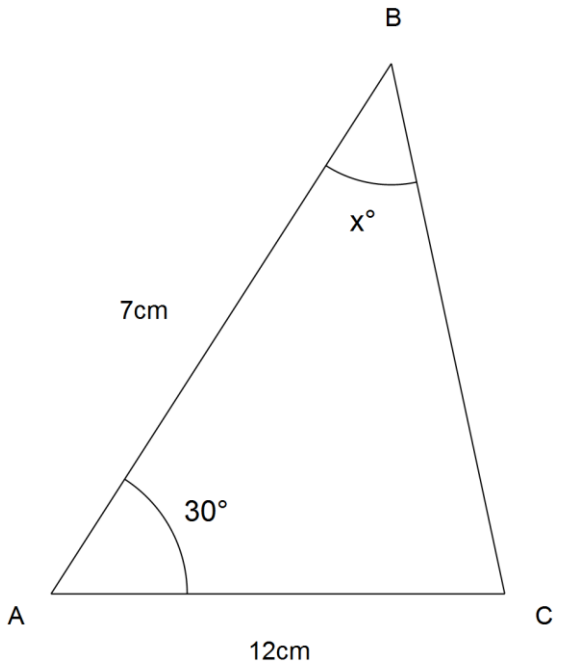


Diagram not

drawn accurately

The diagram shows the triangle ABC and the lengths of two of its sides in centimetres.
Angle BAC=30°.
Find the value of x.

$x =$ .....

(Total 5 marks)

Section B:
Answer either Question 12 which starts below
or Question 13 which is on page 15.
Both are worth 20 marks

12. (a) The point A and B have coordinates (5,1) and (1,7) respectively. Find the equation of the perpendicular bisector of AB.

.....

(3)

(b) (i) $f(x) = x^3 + ax + 2$ has a factor $(x + 2)$. Find the value of a.

$a =$ .....

(2)

(ii) Factorise $f(x)$ fully and sketch a graph of $y = f(x)$.

(4)

(c) Showing your working, find the value of a so that $\frac{2+\sqrt{2}}{\sqrt{18}-4} = \sqrt{a} + \sqrt{a+1}$

$a =$……..

(4)

(d)

(i) Express $x^2 + x + 1$ in the form $(x + b)^2 + c$. Hence sketch the graph $y = x^2 + x + 1$.

(3)

(ii) For what value(s) of m is the line $y = mx$ a tangent to the curve?

$m =$……..

(4)

(Total 20 marks)

13. A *proper factor* of an integer N is a positive integer, not 1 or N, that divides N.
(i) Show that $3^2 \times 5^3$ has exactly 10 proper factors. Determine how many other integers of the form $3^m \times 5^n$ (where m and n are integers) have exactly 10 proper factors.
(ii) Let N be the smallest positive integer that has exactly 426 proper factors. Determine N, giving your answer in terms of its prime factors.

(Total 20 marks)

Use as much space on this and the next page for the working for question 13.

This page is blank and available to be used for working.

END OF PAPER