Name: ……..…..

RADLEY

Academic Scholarship 2018

MATHEMATICS II

Time allowed – 1 hour

Show all working

Calculators can be used

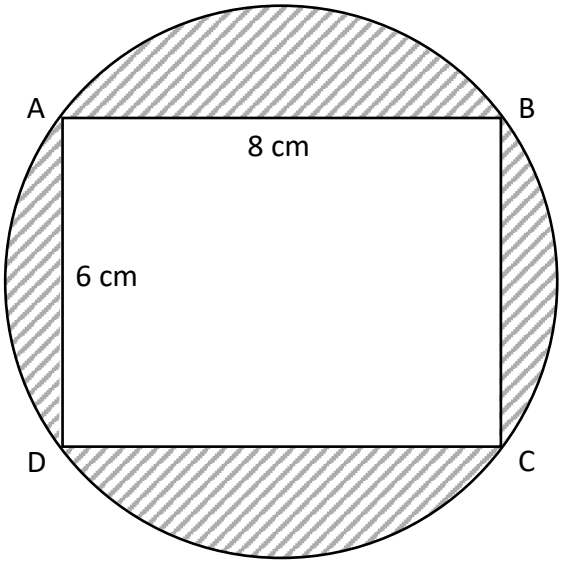
1. A small-holder has a field and a barn. In the field are ewes and lambs. In the barn are geese and turkeys. There is no other live-stock on his small-holding.

   a. In the field are 16 ewes and 25 lambs. What percentage of the animals in the field are lambs?

   b. The farmer sells the 25 lambs. After sales costs of 3.5% have been deducted, the farmer gets £2371.97. What is the price of one lamb before sales costs have been deducted?

   c. 60% of the birds in the barn are turkeys. The farmer sells 20 of these turkeys, and after this sale the percentage of birds remaining in the barn that are turkeys is now 55%. By forming an equation, work out how many geese are in the barn.

2.



   A rectangle, ABCD, in which AB = CD = 8cm, and BC = DA = 6cm, is inscribed inside a circle, as in the diagram above.

   a. Which has the largest perimeter, the rectangle or the circle? Justify your answer.

   b. Calculate the radius of the circle.

   c. Which area is largest, the shaded region or the unshaded rectangle? Justify your answer.

3. Solve the following simultaneous equations. You will get no marks for guessing the answers.

a. $\left.\begin{array}{l} \frac{x}{2} - \frac{y}{3} = \frac{1}{4} \\ \frac{x}{3} - \frac{y}{2} = \frac{1}{3} \end{array}\right\}$

b. $\left.\begin{array}{l} x^2 + y^3 = 22 \\ 2x^2 + 3y^3 = 17 \end{array}\right\}$

c. $\left.\begin{array}{l} x + y = 10 \\ xy = -96 \end{array}\right\}$

4. James and Peter are going to travel from Oxford to Durham, a distance of 400 km. James goes by car, and Peter takes his bicycle. James travels at an average speed of *v* km/h. Peter's average bicycling speed is 55 km/h slower than James's driving speed. They set out together, and Peter arrives in Durham 11 hours after James.

a. Find an expression in terms of *v* for the time it takes James to drive from Oxford to Durham.

b. Find an expression in terms of *v* for the time it takes Peter to bicycle from Oxford to Durham.

c. Write down an equation for *v*.

d. Solve this equation to find the value of *v*.

5. Do not use a calculator for this question

   a. Work out $\left(\frac{2^4-1}{2^2-1}\right) - 1$

   b. Work out $\left(\frac{3^4-1}{3^2-1}\right) - 1$

   c. Work out $\left(\frac{4^4-1}{4^2-1}\right) - 1$

   d. Guess an expression for $\left(\frac{x^4-1}{x^2-1}\right) - 1$

   e. Prove that your expression works for almost all values of *x*.

   f. For which values of *x* does your expression not work?

6. a. Explain why you can write any odd number in the form $2n + 1$, where *n* is an integer.

   b. Prove that the sum of two consecutive odd numbers is always a multiple of four.

   c. Prove that the product of any two consecutive odd numbers always gives a remainder of three when divided by four.

   d. Prove that the product of any three consecutive numbers is always a multiple of six.