

RADLEY

Scholarship Examination

MATHEMATICS II

March 2017

Time allowed – 1 hour

Show all working

Calculators can be used

1. Mario Draghi is sad about Brexit. So he decides to introduce a new pricing structure. On weekdays he will add a set percentage to the normal price of items for sale, and at the weekends he will discount the new price by the same percentage.

He decides to call the percentage the Farage percent:

- a. With a Farage percent of 10%, explain why a bottle of Prosecco that normally costs €10 will cost €11 on weekdays, and €9.90 at weekends.
 - b. Deciding that this is not working he decides to change the Farage percent to 20%. What is the weekend price of a packet of spaghetti that normally costs €2.50?
 - c. He now consults Christine Lagarde. Mrs Lagarde is pleased that she has not been sent to prison, and wants to celebrate by buying an expensive new coat, this weekend. The one she likes has a normal price of €10,000 but she only wants to pay €784. So she persuades her friend, Mr Draghi, to introduce a special Farage percent just for her. What should that percent be?
2. Solve each of the following pairs of simultaneous equations:

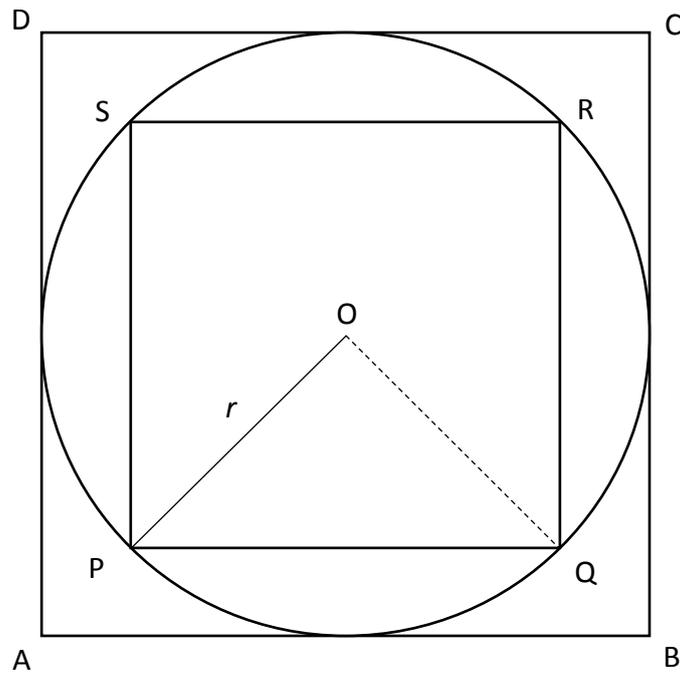
i.
$$\left. \begin{array}{l} 2x + 3y = 14 \\ 3x - 5y = 2 \end{array} \right\}$$

ii.
$$\left. \begin{array}{l} 2\sqrt{x} + 3\sqrt{y} = 14 \\ 3\sqrt{x} - 5\sqrt{y} = 2 \end{array} \right\}$$

ii.
$$\left. \begin{array}{l} 2x + 3y = 14xy \\ 3x - 5y = 2xy \end{array} \right\}$$

3. The bursar buys some shares for £(50x). Later he sells them for £(1800 + 5x). He makes a profit of x%.
 - a. Show that $x^2 + 90x - 3600 = 0$
 - b. Find the value of x.

4.



The diagram shows a square, ABCD.

Inside the square is a circle with centre O and radius r .

Inside the circle is a square, PQRS.

- What is the size of angle POQ?
- Find an expression, in terms of r , for the length PQ.
- Find an expression, in terms of r , for the area of the square PQRS.
- Find an expression, in terms of r , for the length AB.
- Find an expression, in terms of r , for the area of the square ABCD.
- Find the ratio $\text{area}(ABCD) : \text{area}(PQRS)$.

5. Do not use a calculator for this question. Leave your answers as fractions.

a. Show that $\frac{1}{2}\left(\frac{1}{1 \times 2} + \frac{1}{2 \times 3}\right) = \frac{1}{1 \times 3}$

b. Calculate $\frac{1}{2}\left(\frac{1}{2 \times 3} + \frac{1}{3 \times 4}\right)$

c. Calculate $\frac{1}{2}\left(\frac{1}{3 \times 4} + \frac{1}{4 \times 5}\right)$

d. Calculate $\frac{1}{2}\left(\frac{1}{4 \times 5} + \frac{1}{5 \times 6}\right)$

e. Calculate $\frac{1}{2}\left(\frac{1}{5 \times 6} + \frac{1}{6 \times 7}\right)$

f. Write down a formula that summarises all of the above calculations.

g. Justify your formula.

6. Of the following statements, two are true, and two are false. For those that are true, prove why they are true. For those that are false, give an example to show why they are false.

A. If $x > y$, then $x^2 > y^2$.

B. If N is not prime then $N - 2$ is not prime.

C. If n is even, then n^2 is even.

D. For any numbers, x and y , $x^2 + y^2 - 2xy \geq 0$.